Journal of Sound and Vibration (2002) **251**(3), 543–557
doi:10.1006/jsvi.2001.3842, available online at http://www.idealibrary.com on IDEAL®

LETTERS TO THE EDITOR

ACCELERATION-BASED VIBRATION CONTROL OF DISTRIBUTED PARAMETER SYSTEMS USING THE "RECIPROCAL STATE-SPACE FRAMEWORK"

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(*Received 7 September 2000, and in final form 22 June 2001*)

1. INTRODUCTION

Accelerometers represent some of the most widely used sensors in structural systems. This is because displacement and velocity information can be obtained directly by integration which is preferred to differentiation because of noise issues [1]. In addition, transient (shock) motions can be picked up easily, and destructive forces in machinery are more closely related to acceleration than they are to displacement and velocity [2]. A number of studies have attempted to implement acceleration feedback indirectly (via integration and estimation) or in other methodologies [3–8]. Most of the studies did not try to design *acceleration-based feedback controllers* based on full-state feedback control schemes. In traditional state-space techniques, acceleration data are integrated to obtain position and velocity information. However, when a limited number of accelerometers are attached to a structure (normally a distributed system), position and velocity information, if integrated, is only obtained at the point where the accelerometer is attached. The total state that is distributed throughout the whole structure cannot be obtained by simply integrating the measured acceleration information. Therefore, an observer is usually built to estimate the other missing states. However, there exist several problems in designing a state estimator with acceleration measurements. These issues will be explained in detail later.

In this study, a direct full-state-derivative feedback control scheme is developed using the "reciprocal state-space" (RSS) methodology which was informally introduced in 1997 by Tseng and Yedavalli [9]. The RSS framework hinges on mathematically switching the state vector with the derivative of the state vector. Thus, the controller and estimators utilize the derivative of the state vector, instead of the state vector itself. RSS was simulated for an aircraft roll maneuver control problem by Kwak and co-workers [10, 11]. A simulation was

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also accomplished in the RSS for vibration dissipation of aircraft landing gear components [12]. In this paper, the theoretical basis is explained and experimentally validated for the first time. The framework is novel because it enables one to implement full-state feedback control with a state-derivative estimator using acceleration information directly. Additional novelty lies in the fact that this is the first time that the linear quadratic regulator (LQR) controller design scheme, with non-standard performance indices, is applied to the state-derivative feedback controller and estimator. Finite element modelling, orthogonal modal co-ordinate transformations, and modal reduction techniques are also used to build the actual model.

2. RECIPROCAL STATE-SPACE FRAMEWORK

The dynamics of a structural system can be expressed in the form of a finite dimensional, multivariable, "matrix second order (MSO)" differential equation through the well-known finite element method [10, 11, 13–17] as shown in equation (1) [10, 11, 14–17]:

$$M\ddot{q} + C\dot{q} + Kq = Bf, \tag{1}$$

where $M_{n\times n}$, $C_{n\times n}$ and $K_{n\times n}$ are the mass, damping, and stiffness matrices respectively. $B_{n\times n}$ is an actuator distribution matrix. $q_{n\times 1}$ and $f_{m\times 1}$ are the generalized co-ordinate and the external forcing vector respectively. In order to design controllers and estimators for various systems, the matrix second order system can be converted into standard first order state-space framework:

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} f, \tag{2}$$

where $x = [q\ \dot{q}]^{\mathrm{T}}$ and $u = f$. Full-state feedback control in the "state space" is then written as

$$\dot{x} = Ax + Bu, \qquad u = -Lx. \tag{3, 4}$$

The closed-loop system is then given as

$$\dot{x} = (A - BL)x. \tag{5}$$

When examining equations (2)–(5), from the point of view of vibration control, the state variable, x, consists of only displacement and velocity terms. In many structural applications, however, accelerometers are the sensors of choice. This can present problems for direct measurement. For example, when an accelerometer is attached at the tip of a cantilevered beam as shown in Figure 1. The measured data from the accelerometer represent only the tip acceleration. This means that only the *tip velocity and displacement* can be obtained by integrating the data. Therefore, one may need to build an observer in order to obtain full-state information. A typical observer presents its own problems as elucidated in this simple demonstration. The equations for a typical first order state observer are given in equations (6)–(9):

$$\dot{\hat{x}} = A\hat{x} + Bu + F(y(t) - \hat{y}(t)), \tag{6}$$

$$y(t) = Cx, \qquad \hat{y}(t) = C\hat{x}, \qquad u = -L\hat{x}. \tag{7–9}$$

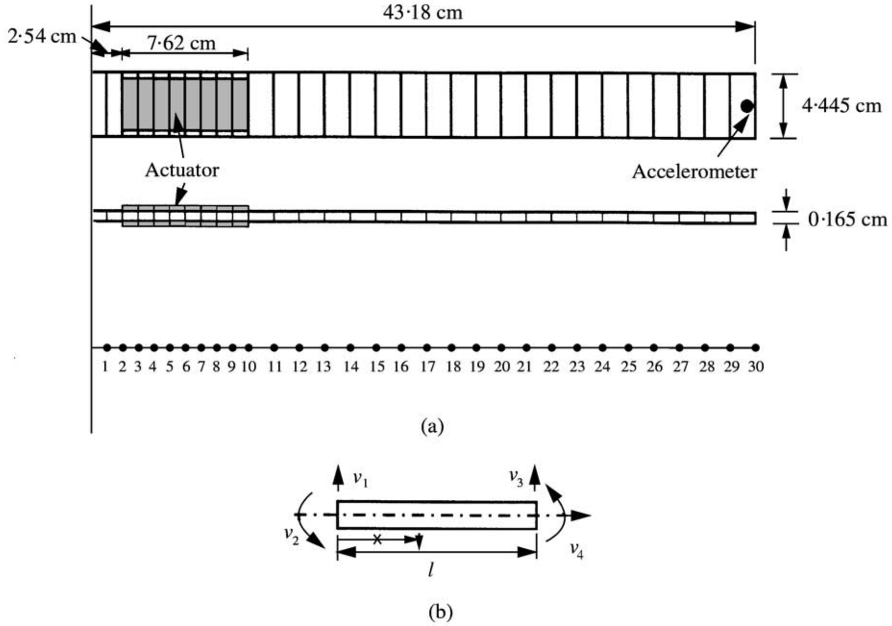


Figure 1. (a) Finite element model of the cantilevered beam. (b) A beam bending element.

From equation (7), it is easily seen that there is *no acceleration* information because the state x contains only velocity and displacement information. Therefore, the observability matrix [18] is singular because the output matrix C is empty. As a result, this system cannot be observable. Let us now examine the same problem from the perspective of output feedback. Traditionally, the output feedback control technique has been used for acceleration-based measurements. Output feedback control, however, is more cumbersome and impractical [19]. The control input and closed-loop system for an output feedback controller are given as

$$u = -Ly = -LCx, \qquad \dot{x} = (A - BKC)x. \tag{10, 11}$$

The major difficulty with output feedback stems from the fact that the dimension of the output feedback gain L is $m \times p$, while that of state feedback gain is $m \times n$. Since p is generally less than n, one can manipulate fewer scalar control gains in output feedback design than in state feedback design. Therefore, pole placement, eigenstructure assignment, and optimal control design are in general more difficult to accomplish that using state feedback. For example, in optimal output feedback control, three non-linear coupled equations shown in equations (12)–(14) need to be solved in order to find the optimal feedback gain(s):

$$0 = SA_c + A_c^{\mathrm{T}}S + C^{\mathrm{T}}L^{\mathrm{T}}RL^{\mathrm{T}}C + Q, \tag{12}$$

$$0 = PA_c + A_c^{\mathrm{T}}P + X, \quad X = E\{x(0)x^{\mathrm{T}}(0)\}, \tag{13}$$

$$L = R^{-1}B^{\mathrm{T}}SPC^{\mathrm{T}}(CPC^{\mathrm{T}})^{-1}. \tag{14}$$

Steven and Lewis [19] suggested several numerical approaches to obtain the optimal gain L. However, there is no guarantee for convergence.

In this study, a new control design technique named full-state-derivative control based on the RSS framework [9–12, 20] is introduced to overcome these problems as shown in equations (15) and (16):

$$x = G\dot{x} + Hu, \qquad u = -L\dot{x}, \tag{15, 16}$$

where

$$G = A^{-1}, \qquad H = -A^{-1}B. \tag{17, 18}$$

Note that the eigenvalues of the system matrix G in the reciprocal state space are the reciprocal of those in the standard state-space framework. Thus, the stability conditions for both the reciprocal state space and the standard state-space frameworks are the same. Now, a state-derivative observer in the RSS is written as

$$\hat{x} = G\dot{\hat{x}} + Hu + F(y(t) - \hat{y}(t)) \tag{19}$$

$$y(t) = C\dot{x}, \qquad \hat{y}(t) = C\dot{\hat{x}}, \qquad u = -L\dot{\hat{x}}. \tag{20–22}$$

TABLE 1

Comparison of standard state space and RSS for acceleration measurements

		Standard state space	RSS
Framework		$\dot{x} = Ax + Bu$ $u = -Lx$ State feedback	$x = G\dot{x} + Hu$ $u = -L\dot{x}$ State-derivative feedback
Observer		$\dot{\hat{x}} = A\hat{x} + Bu + F(y(t) - \hat{y}(t))y(t) = Cx$ $\hat{y}(t) = C\hat{x}$ $u = -L\hat{x}$ *No acceleration information* in the state x (unobservable)	$\hat{x} = G\dot{\hat{x}} + Hu + F(y(t) - \hat{y}(t))$ $y(t) = C\dot{x}$ $\hat{y}(t) = C\dot{\hat{x}}$ $u = -L\dot{\hat{x}}$ *Acceleration information* in the state-derivative $\dot{x}$ (observable)
Optimal control (LQR)	Performance index	$J = \int_0^\infty (x^T Q x + u^T R u)\,dt$ Minimize the states	$J = \int_0^\infty (\dot{x}^T Q \dot{x} + u^T R u)\,dt$ Minimize the state derivatives
	Feedback	$y = C\dot{x}$ $u = -Ly$ $L: m \times p$ *Output feedback*	$u = L\dot{x}$ $L: m \times n$ $n \geqslant p$ *State-derivative feedback*
	Control gain	$0 = SA_c + A_c^T S + C^T L^T R L^T C + Q$ $0 = PA_c + A_c^T P + X,\ X = E\{x(0)x^T(0)\}$ $L = R^{-1}B^T SPC^T(CPC^T)^{-1}$ Complex coupled equations; *Impractical*	$0 = SG + G^T S - SHR^{-1}H^T S + Q$ $L = R^{-1}H^T S$ Simple algebraic Riccati equation; *Practical*
	Closed loop	$A_c = A - BKC$	$x = (G - HL)\dot{\hat{x}} = G_c\dot{\hat{x}}$

As shown in equation (20), the state derivative $\dot{x}$, which has acceleration information, can be used directly in the state-derivative estimator in the RSS. Moreover, according to references [9–12, 20], most controller design techniques in the state-space framework like pole placement and LQR can be directly applied to the RSS framework. The closed-loop system in the RSS framework is given as

$$x = (G - HL)\dot{\hat{x}} = G_c\dot{\hat{x}}, \tag{23}$$

where G_c is a closed-loop system matrix. The overall comparison of the standard state-space framework and the RSS is shown in Table 1.

3. OPTIMAL CONTROL DESIGN USING NON-STANDARD PERFORMANCE INDICES

In the RSS framework, the state derivatives, not the states themselves, are directly controlled. In other words, the state derivatives are minimized in an optimal fashion, when RSS is applied to optimal control. The state-derivative performance index to be minimized is written as

$$J_{\dot{x}} = \int_0^\infty \dot{x}^{\mathrm{T}}\dot{x}\,\mathrm{d}t. \tag{24}$$

By adding a control performance index, one gets the non-standard form

$$J = \int_0^\infty (\dot{x}^{\mathrm{T}}Q\dot{x} + u^{\mathrm{T}}Ru)\,\mathrm{d}t. \tag{25}$$

This non-standard performance index makes the state-derivative feedback controller design easy when compared to the standard output feedback in the state-space framework. It is shown that output feedback control is more complicated than state feedback control [18–20]. However, if we use the RSS framework, finding an optimal output feedback gain L becomes quite straightforward. The control input in the RSS shown in equation (22) is given as

$$u = -L\dot{x}. \tag{26}$$

In this framework, the closed-loop system becomes

$$x = (G - HL)\dot{x} = G_c\dot{x}. \tag{27}$$

Finding the L which minimizes J in this new system description is much easier because we can use the standard parameter optimization methodology to find the gains involved in the Lyapunov matrix. By substituting equation (26) into equation (25), the performance index can be expressed as

$$J = \int_0^\infty [\dot{x}^{\mathrm{T}}Q\dot{x} + (L\dot{x})^{\mathrm{T}}R(L\dot{x})]\,\mathrm{d}t = \int_0^\infty \dot{x}^{\mathrm{T}}(Q + L^{\mathrm{T}}RL)\dot{x}\,\mathrm{d}t. \tag{28}$$

Suppose that one can find a constant, positive-semi-definite matrix S which satisfies [18, 21]

$$\frac{\partial}{\partial t}(x^{\mathrm{T}}Sx) = -\dot{x}^{\mathrm{T}}(Q + L^{\mathrm{T}}RL)\dot{x}. \tag{29}$$

Since $x = G_c \dot{x}$,

$$\frac{\partial}{\partial t}(x^T S x) = \dot{x}^T (S G_c + G_c^T S)\,\dot{x}. \tag{30}$$

Then, one may rewrite equations (29) and (30) as

$$0 = S G_c + G_c^T S - L^T R L + Q \tag{31}$$

or

$$0 = SG + G^T S - SHR^{-1} H^T S + Q. \tag{32}$$

The above equation is the RSS version of the well-known algebraic Ricatti equation. The corresponding feedback gain is expressed as

$$L = R^{-1} H^T S. \tag{33}$$

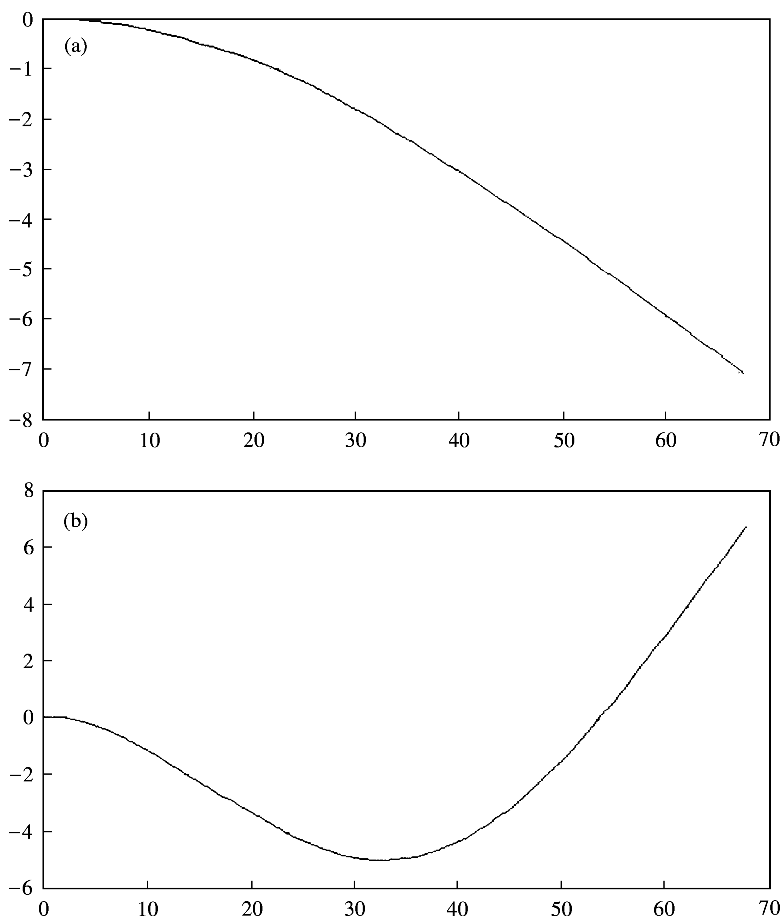


Figure 2. Frequencies and mode shapes of FEM: (a) mode 1 (7·6638 Hz); (b) mode 2 (45·1662 Hz). WAM/95/00.

4. ILLUSTRATIVE EXAMPLE

In this section, a new observer based, RSS controller design scheme for vibration dissipation of a cantilevered aluminium beam with piezoelectric actuators and an accelerometer at the tip is presented (Figure 1). The dimensions of the beam and the PZT actuators are 43·18 cm × 4·445 cm × 0·165 cm and 7·62 cm × 3·81 cm × 0·025 cm respectively. The fundamental frequency of the beam is experimentall(y measured at 7·58 Hz. The finite element model (FEM) procedure is employed to analyze the dynamics of the system. The model consists of "39" multi-layered composite two-node bending elements as shown in Figure 1. Figure 2 shows the mode shapes and frequencies of the FEM. The fundamental frequency of the FEM is determined to be 7·65 Hz which matches well with the experimental result.

4.1. FINITE ELEMENT MODEL

The FEM procedure for piezoelectric actuators attached to composite structures is shown in references [22–25]. For actuator designs, the variational method and virtual work are applied. The actuator distribution matrix can be written as

$$K_{sp} = -\int_0^l Y b_a d_{31} h_a [N'']^{\mathrm{T}} [N_E] \, \mathrm{d}x, \tag{34}$$

where the structural and electrical shape functions $[N]$ and $[N_E]$ are given in reference [12]. The final equation of motion of the system is written as

$$(M_s + M_p)\ddot{q} + (K_s + K_p)q = K_{sp}E_f, \tag{35}$$

where

$$M_s = \rho_s A_s \int_0^l [N]^{\mathrm{T}}[N] \, \mathrm{d}x, \qquad M_p = \rho_p A_p \int_0^l [N]^{\mathrm{T}}[N] \, \mathrm{d}x, \tag{36, 37}$$

$$K_s = E_s I_s \int_0^l [N'']^{\mathrm{T}}[N''] \, \mathrm{d}x, \qquad K_p = E_p I_p \int_0^l [N'']^{\mathrm{T}}[N''] \, \mathrm{d}x. \tag{38, 39}$$

ρ, A, E, I, b, h, and d_{31} are the mass density, the cross-sectional area, Young's modulus, the moment of inertia, the width, the distance from the mid-plane and the piezoelectric constant respectively. The subscripts s and p are denoted as the structure and the piezoelectric material respectively. The measurement equation is expressed as

$$E_m = K_{sen} q, \tag{40}$$

where

$$K_{sen} = [0 \;\; 0 \;\; \cdots \;\; a_s] \tag{41}$$

and a_s is the sensitivity of the accelerometer. Since the size of system shown in equation (35) is too large (60 × 60) and the first mode is the only one that we are concerned with, system size reduction is required.

4.2. MODEL REDUCTION IN ORTHOGONAL MODAL CO-ORDINATES

The dimension of the discretized continuous system shown in equation (35) depends on the number of elements and the degrees of freedom of each element. For most physical systems, the dimensions are usually large. A large system size, however, is not desirable from a computational perspective. In addition, we are interested in a limited number of dominant modes. Thus, a system size reduction technique is required. In order to reduce the system size, a modal co-ordinate transformation is applied [13]. The modal equation of motion of the system can be obtained with a similarity transformation that diagonalizes the integrated stiffness matrix [14]. Let

$$q = T\eta, \tag{42}$$

where $T = [t_1\ t_2\ t_3 \cdots t_n]$ and t_i is the eigenvector of the system given in equation (42). The eigenvectors t_i are mutually orthogonal with respect to the mass, M, and stiffness, K, matrices; η is the modal co-ordinate. Introducing equation (42) into equation (35) yields

$$\ddot{\eta} + C_\eta \dot{\eta} + \Lambda \eta = F_\eta E_f, \qquad y = K_{sen_\eta} \eta, \tag{43, 44}$$

where C_η is the modal damping matrix and

$$\eta = [\eta_1\ \eta_2\ \eta_3\ \cdots\ \eta_n]^{\mathrm{T}}, \qquad T^{\mathrm{T}}(M_s + M_p)T = I, \tag{45, 46}$$

$$\Lambda = T^{-1}(K_s + K_p)T = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n), \tag{47}$$

$$F_\eta = T^{\mathrm{T}} K_{sp}, \qquad K_{sen_\eta} = K_{sen} T. \tag{48, 49}$$

The eigenvalues λ_i are real numbers. Now, the system size can be reduced by truncating the higher modes. After the system size is reduced, the system is transformed to the RSS

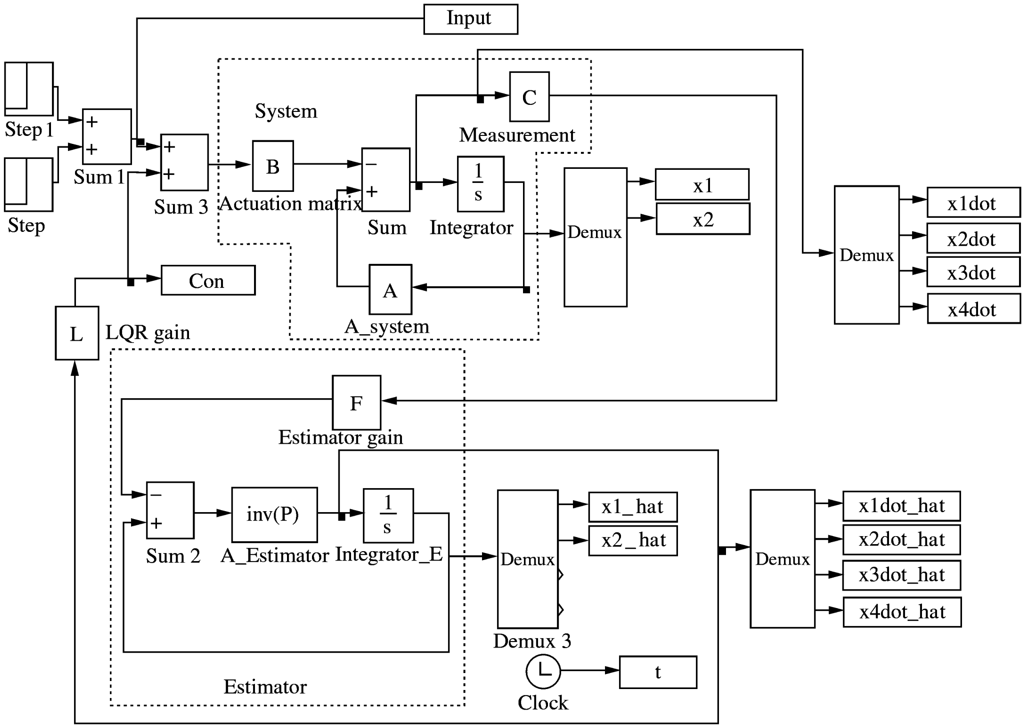


Figure 3. Simulation diagram in Simulink.

framework. The RSS representation of the reduced model is expressed as

$$\begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -\Lambda^{-1}C_\eta & -\Lambda^{-1} \\ I & 0 \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix} + \begin{bmatrix} -\Lambda^{-1}K_{sp_\eta} \\ 0 \end{bmatrix} E_f, \tag{50}$$

$$y = [0,\ K_{sen_\eta}] \begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix}, \quad x(t) = G\dot{x}(t) + Hu(t), \quad y(t) = C\dot{x}(t). \tag{51–53}$$

A LQR with non-standard performance indices is employed to build a full-state-derivative feedback controller and an observer. A state-derivative observer is also built to implement the full-state-derivative feedback control. The controller and estimator of the system are given by the following:

$$x = G\dot{x} + Hu, \qquad \hat{x} = G\dot{\hat{x}} + Hu + F(y(t) - \hat{y}(t)), \tag{54, 55}$$

$$y(t) = C\dot{x}, \qquad \hat{y}(t) = C\dot{\hat{x}}, \qquad u = -L\dot{\hat{x}}. \tag{56–58}$$

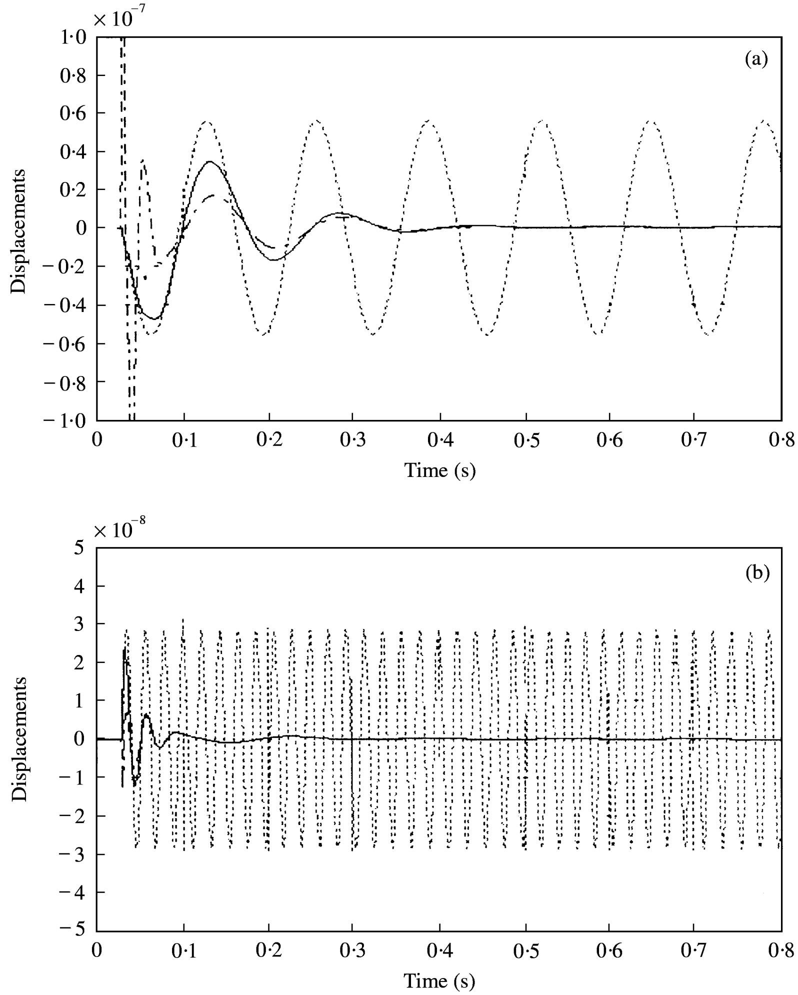


Figure 4. Time responses for the displacements, impulse response at (200 V): (a) first mode and (b) second mode; - - - -, open loop; ——, closed loop; – · –, estimates.

For the LQR design technique, the non-standard performance indices to be minimized for both controller and estimator are expressed as

$$J = \int_0^{\infty} [\dot{x}^{\mathrm{T}} Q \dot{x} + u^{\mathrm{T}} R u]\,\mathrm{d}t, \qquad J_e = \int_0^{\infty} [\dot{e}^{\mathrm{T}} Q_e \dot{e} + y^{\mathrm{T}} R_e y]\,\mathrm{d}t, \tag{59, 60}$$

where Q and Q_e are positive semi-definite. R and R_e are positive definite and $e = [x - \hat{x}]$. The subscript e is denoted as the estimator. According to references [9, 19, 21], the LQR state-derivative feedback gain and estimator gain are written as

$$L = R^{-1} H^{\mathrm{T}} S, \qquad F^{\mathrm{T}} R_e^{-1} C^{\mathrm{T}} S_e. \tag{61, 62}$$

The matrix S and S_e can be obtained from the associated algebraic matrix Riccati equations as shown in equation (32). The closed-loop system is written as

$$x = G\dot{x} - HL\dot{\hat{x}}, \qquad \hat{x} = G\dot{\hat{x}} - HL\dot{\hat{x}} + FC\dot{x} - FC\dot{\hat{x}}. \tag{63, 64}$$

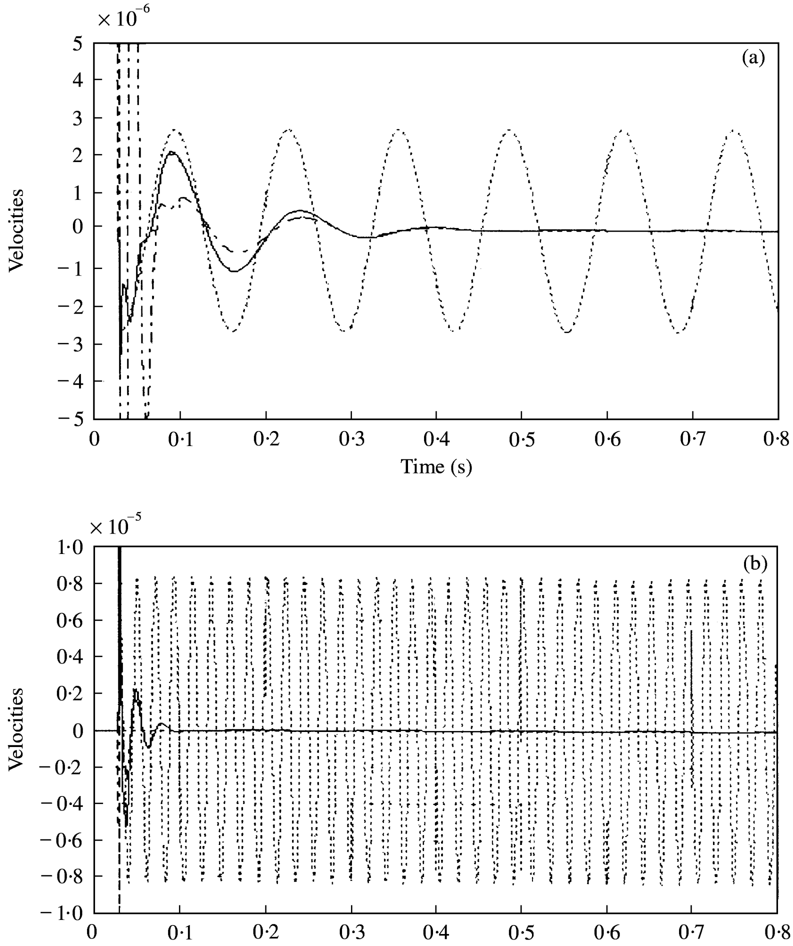


Figure 5. Time responses for the velocity, impulse response at (200 V): (a) first mode and (b) second mode; - - - -, open loop; ——, closed loop; – · –, estimates.

For implementation, the equations need to be reorganized in the form of standard state-space representation after the optimal full-state-derivative gain is obtained. Equations (63) and (64) are rewritten as

$$\dot{x} = G^{-1}x + G^{-1}HL\dot{\hat{x}} = Ax - BL\dot{\hat{x}}, \qquad \dot{\hat{x}} = P^{-1}\hat{x} - P^{-1}FC\dot{x}, \qquad (65, 66)$$

where $P = G - HL - FC$. The simulation diagram of the equation is shown in Figure 3. According to the figure, the full-state-derivative feedback control scheme is clearly shown. Since only the acceleration information is measured, the first column of the C matrix is zero. A full order estimator is built to obtain velocity estimates. The estimates of acceleration and velocity information can be used as feedback signals in the RSS framework. Figures 4–6 show the impulse responses for displacement, velocity, and acceleration. According to Figures 4–6, the RSS based controller and estimator with acceleration feedback can be used to dissipate vibration of the system. The impulse input and control input are given in Figure 7. For experimental implementation, the initial condition response is selected.

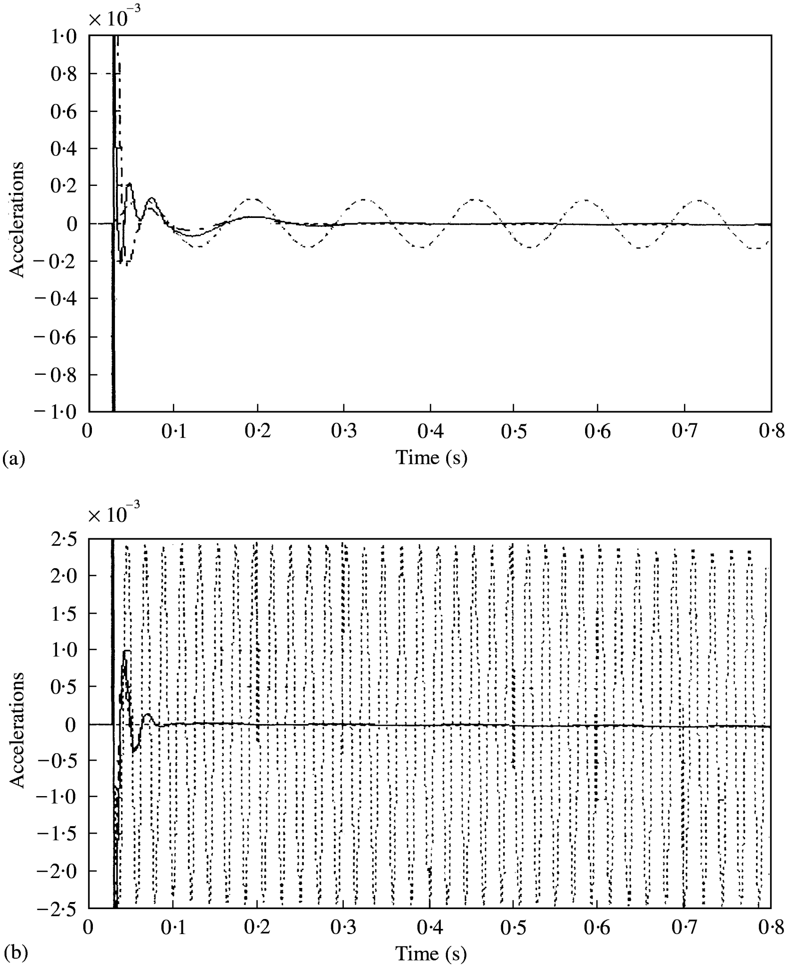
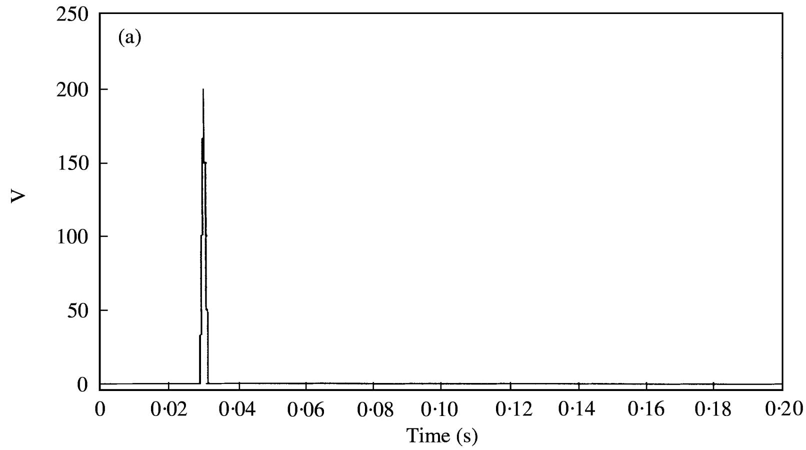


Figure 6. Time responses for the acceleration, impulse response at (200 V): (a) first mode and (b) second mode; - - - -, open loop; ——, closed loop; – · –, estimates.



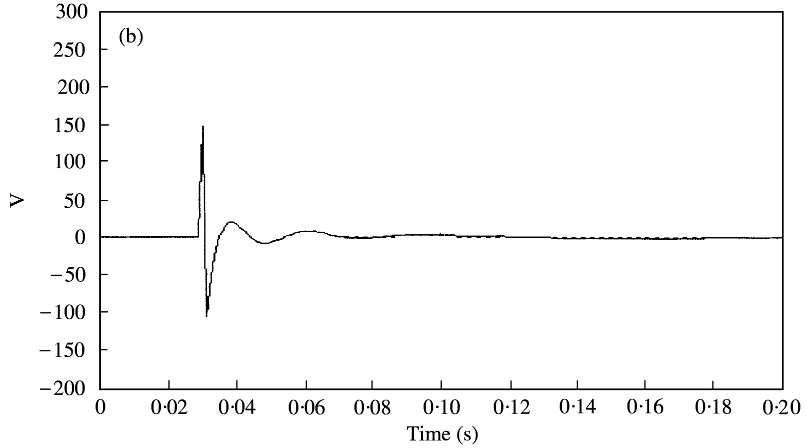


Figure 7. Inputs: (a) impulse input and (b) control input.

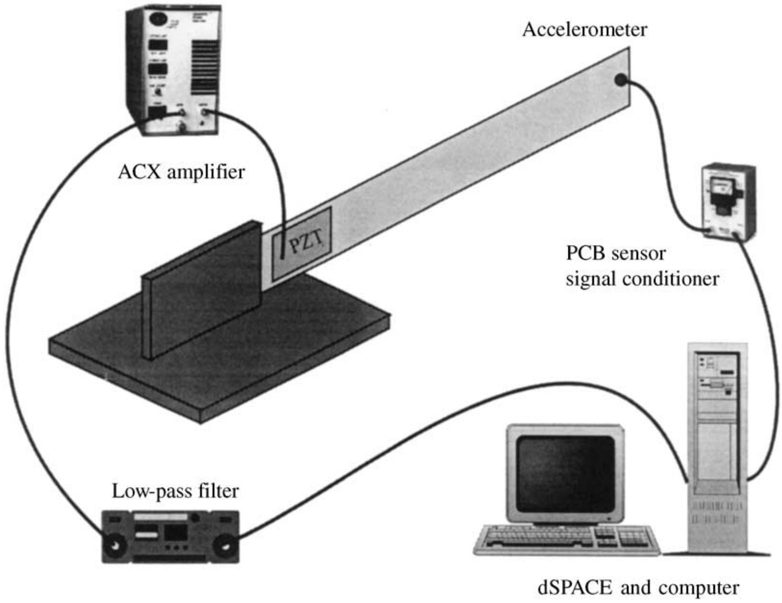


Figure 8. Actual experimental set-up.

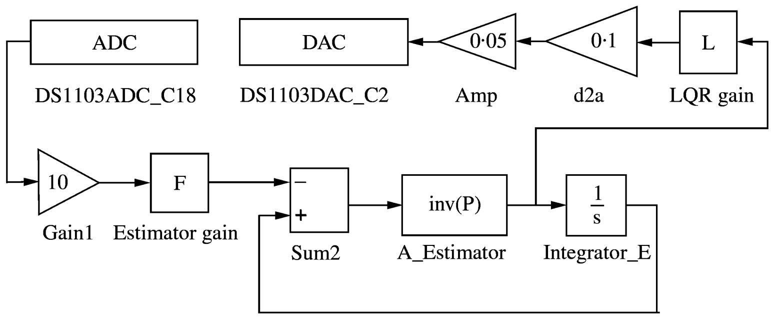


Figure 9. dSPACE diagram.

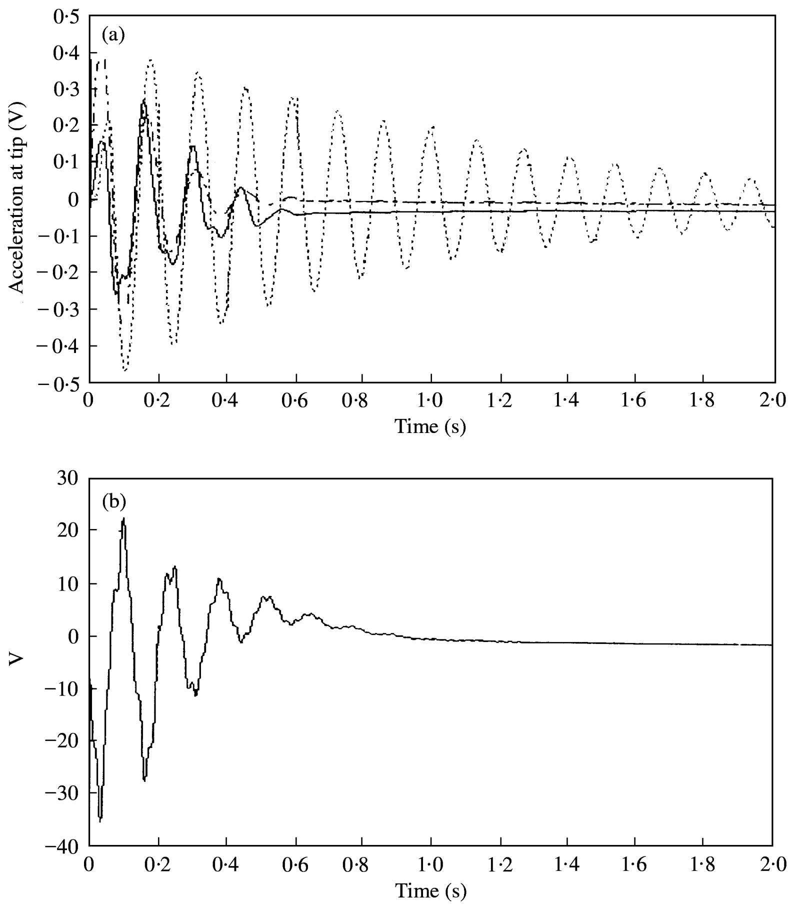


Figure 10. Experimental result (initial input): (a) tip acceleration, - - - -, open loop; ——, closed loop; – · –, estimates; and (b) control input.

The actual experimental set-up of the vibration control system is shown in Figure 8. A dSPACE data acquisition and control unit is employed. A low-pass filter is used to eliminate high-frequency digital noise. The dSPACE control diagram is given in Figure 9. Figure 10 shows the actual tip acceleration of the beam for both open-loop and closed-loop cases and control input versus time. The dotted line shows the open-loop responses and the solid line represents the closed-loop responses. According to the result, the controller in the RSS absorbs the vibration in 0·5 s.

5. CONCLUSIONS AND FUTURE WORK

In this study, a full-state-derivative feedback control with state-derivative estimator using an acceleration measurement is presented. The "reciprocal state space" (RSS) framework is employed to implement the control. The framework is very attractive because acceleration measurements can be directly fed back, which was not possible in the standard state-space framework. This methodology is not meant to be a replacement for the standard state-space formulation in all systems, but in systems where accelerometers are the only sensors used, it can be quite useful. When accelerometers are used, the RSS technique has many advantages over other standard state-space techniques. This is expected since there is a strong relationship between the actual measurement and the state-derivative vector. This paper does not present a methodology that will give better results for a particular system. It does, however, present a methodology that will enable one to implement vibration control from accelerometer data in a simpler fashion than what is currently available.

A PZT laminated cantilevered steel beam with one accelerometer was chosen to be a model. The finite element model procedure, orthogonal modal co-ordinate transformation and modal reduction techniques were employed to complete the model. The LQR design method with performance indices, formulated in the RSS framework, was used to design both the controller and the estimator. Simulations and experimental implementation of the full-state-derivative controller based on acceleration measurements are given. According to the simulations and experiment, direct acceleration feedback can be used in the reciprocal state space to achieve acceptable results.

In this study, the first mode is only considered as a simple example to validate the methodology. Multiple vibration modes will be controlled in future. Controllers and estimators also will be designed using pole placement and eigenstructure assignment techniques in the RSS framework.

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